

CHAPTER 23

NUCLEAR CHEMISTRY

23.5 (a) The atomic number sum and the mass number sum must remain the same on both sides of a nuclear equation. On the left side of this equation the atomic number sum is 13 ($12 + 1$) and the mass number sum is 27 ($26 + 1$). These sums must be the same on the right side. The atomic number of X is therefore 11 ($13 - 2$) and the mass number is 23 ($27 - 4$). X is sodium-23 (${}^{23}_{11}\text{Na}$).

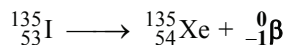
(b) X is ${}^1_1\text{H}$ or ${}^1_1\text{p}$ (d) X is ${}^{56}_{26}\text{Fe}$

(c) X is ${}^1_0\text{n}$ (e) X is ${}^0_{-1}\beta$

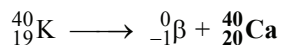
23.6 **Strategy:** In balancing nuclear equations, note that the sum of atomic numbers and that of mass numbers must match on both sides of the equation.

Solution:

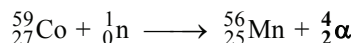
(a) The sum of the mass numbers must be conserved. Thus, the unknown product will have a mass number of 0. The atomic number must be conserved. Thus, the nuclear charge of the unknown product must be -1 . The particle is a β particle.



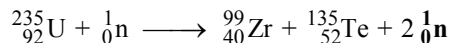
(b) Balancing the mass numbers first, we find that the unknown product must have a mass of 40. Balancing the nuclear charges, we find that the atomic number of the unknown must be 20. Element number 20 is calcium (Ca).



(c) Balancing the mass numbers, we find that the unknown product must have a mass of 4. Balancing the nuclear charges, we find that the nuclear charge of the unknown must be 2. The unknown particle is an alpha (α) particle.



(d) Balancing the mass numbers, we find that the unknown products must have a combined mass of 2. Balancing the nuclear charges, we find that the combined nuclear charge of the two unknown particles must be 0. The unknown particles are neutrons.



23.13 We assume the nucleus to be spherical. The mass is:

$$235 \text{ amu} \times \frac{1 \text{ g}}{6.022 \times 10^{23} \text{ amu}} = 3.90 \times 10^{-22} \text{ g}$$

The volume is, $V = 4/3\pi r^3$.

$$V = \frac{4}{3}\pi \left((7.0 \times 10^{-3} \text{ pm}) \times \frac{1 \text{ cm}}{1 \times 10^{10} \text{ pm}} \right)^3 = 1.4 \times 10^{-36} \text{ cm}^3$$

The density is:

$$\frac{3.90 \times 10^{-22} \text{ g}}{1.4 \times 10^{-36} \text{ cm}^3} = 2.8 \times 10^{14} \text{ g/cm}^3$$

23.14 Strategy: The principal factor for determining the stability of a nucleus is the *neutron-to-proton ratio* (n/p). For stable elements of low atomic number, the n/p ratio is close to 1. As the atomic number increases, the n/p ratios of stable nuclei become greater than 1. The following rules are useful in predicting nuclear stability.

- 1) Nuclei that contain 2, 8, 20, 50, 82, or 126 protons or neutrons are generally more stable than nuclei that do not possess these numbers. These numbers are called *magic numbers*.
- 2) Nuclei with even numbers of both protons and neutrons are generally more stable than those with odd numbers of these particles (see Table 23.2 of the text).

Solution:

- (a) **Lithium-9** should be less stable. The neutron-to-proton ratio is too high. For small atoms, the n/p ratio will be close to 1:1.
- (b) **Sodium-25** is less stable. Its neutron-to-proton ratio is probably too high.
- (c) **Scandium-48** is less stable because of odd numbers of protons and neutrons. We would not expect calcium-48 to be stable even though it has a magic number of protons. Its n/p ratio is too high.

23.15 Nickel, selenium, and cadmium have more stable isotopes. All three have even atomic numbers (see Table 23.2 of the text).

- 23.16**
- (a) **Neon-17** should be radioactive. It falls below the belt of stability (low n/p ratio).
 - (b) **Calcium-45** should be radioactive. It falls above the belt of stability (high n/p ratio).
 - (c) All **technetium** isotopes are radioactive.
 - (d) **Mercury-195** should be radioactive. Mercury-196 has an even number of both neutrons and protons.
 - (e) All **curium** isotopes are unstable.

23.17 The mass change is:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{-436400 \text{ J/mol}}{(3.00 \times 10^8 \text{ m/s})^2} = -4.85 \times 10^{-12} \text{ kg/mol H}_2$$

Is this mass measurable with ordinary laboratory analytical balances?

23.18 We can use the equation, $\Delta E = \Delta mc^2$, to solve the problem. Recall the following conversion factor:

$$1 \text{ J} = \frac{1 \text{ kg} \cdot \text{m}^2}{\text{s}^2}$$

The energy loss in one second is:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{5 \times 10^{26} \text{ kg} \cdot \text{m}^2}{1 \text{ s}^2} \left(\frac{1}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2} \right) = 6 \times 10^9 \text{ kg}$$

Therefore the rate of mass loss is $6 \times 10^9 \text{ kg/s}$.

23.19 We use the procedure shown in Example 23.2 of the text.

(a) There are 4 neutrons and 3 protons in a Li-7 nucleus. The predicted mass is:

$$(3)(\text{mass of proton}) + (4)(\text{mass of neutron}) = (3)(1.007825 \text{ amu}) + (4)(1.008668 \text{ amu})$$

$$\text{predicted mass} = 7.058135 \text{ amu}$$

The mass defect, that is the difference between the predicted mass and the measured mass is:

$$\Delta m = 7.01600 \text{ amu} - 7.058135 \text{ amu} = -0.042135 \text{ amu}$$

The mass that is converted in energy, that is the energy released is:

$$\Delta E = \Delta mc^2 = \left(-0.042135 \text{ amu} \times \frac{1 \text{ kg}}{6.022 \times 10^{26} \text{ amu}} \right) (3.00 \times 10^8 \text{ m/s})^2 = -6.30 \times 10^{-12} \text{ J}$$

The nuclear binding energy is $6.30 \times 10^{-12} \text{ J}$. The binding energy per nucleon is:

$$\frac{6.30 \times 10^{-12} \text{ J}}{7 \text{ nucleons}} = 9.00 \times 10^{-13} \text{ J/nucleon}$$

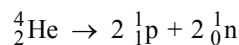
Using the same procedure as in (a), using 1.007825 amu for ${}^1_1\text{H}$ and 1.008665 amu for ${}^1_0\text{n}$, we can show that:

(b) For chlorine-35: Nuclear binding energy = $4.92 \times 10^{-11} \text{ J}$
 Nuclear binding energy per nucleon = $1.41 \times 10^{-12} \text{ J/nucleon}$

23.20 Strategy: To calculate the nuclear binding energy, we first determine the difference between the mass of the nucleus and the mass of all the protons and neutrons, which gives us the mass defect. Next, we apply Einstein's mass-energy relationship [$\Delta E = (\Delta m)c^2$].

Solution:

(a) The binding energy is the energy required for the process



There are 2 protons and 2 neutrons in the helium nucleus. The mass of 2 protons is

$$(2)(1.007825 \text{ amu}) = 2.015650 \text{ amu}$$

and the mass of 2 neutrons is

$$(2)(1.008665 \text{ amu}) = 2.017330 \text{ amu}$$

Therefore, the predicted mass of ${}^4_2\text{He}$ is $2.015650 + 2.017330 = 4.032980 \text{ amu}$, and the mass defect is

$$\Delta m = 4.032980 \text{ amu} - 4.0026 \text{ amu} = 0.0304 \text{ amu}$$

The energy change (ΔE) for the process is

$$\Delta E = (\Delta m)c^2$$

$$= (0.0304 \text{ amu})(3.00 \times 10^8 \text{ m/s})^2$$

$$= 2.74 \times 10^{15} \frac{\text{amu} \cdot \text{m}^2}{\text{s}^2}$$

Let's convert to more familiar energy units (J/He atom).

$$\frac{2.74 \times 10^{15} \text{amu} \cdot \text{m}^2}{1 \text{s}^2} \times \frac{1.00 \text{g}}{6.022 \times 10^{23} \text{amu}} \times \frac{1 \text{kg}}{1000 \text{g}} \times \frac{1 \text{J}}{\frac{1 \text{kg} \cdot \text{m}^2}{\text{s}^2}} = 4.55 \times 10^{-12} \text{J}$$

This is the nuclear binding energy. It's the energy required to break up one helium-4 nucleus into 2 protons and 2 neutrons.

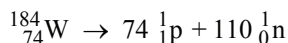
When comparing the stability of any two nuclei we must account for the fact that they have different numbers of nucleons. For this reason, it is more meaningful to use the *nuclear binding energy per nucleon*, defined as

$$\text{nuclear binding energy per nucleon} = \frac{\text{nuclear binding energy}}{\text{number of nucleons}}$$

For the helium-4 nucleus,

$$\text{nuclear binding energy per nucleon} = \frac{4.55 \times 10^{-12} \text{J/He atom}}{4 \text{ nucleons/He atom}} = 1.14 \times 10^{-12} \text{J/nucleon}$$

(b) The binding energy is the energy required for the process



There are 74 protons and 110 neutrons in the tungsten nucleus. The mass of 74 protons is

$$(74)(1.007825 \text{amu}) = 74.57905 \text{amu}$$

and the mass of 110 neutrons is

$$(110)(1.008665 \text{amu}) = 110.9532 \text{amu}$$

Therefore, the predicted mass of ${}_{74}^{184}\text{W}$ is $74.57905 + 110.9532 = 185.5323 \text{amu}$, and the mass defect is

$$\Delta m = 185.5323 \text{amu} - 183.9510 \text{amu} = 1.5813 \text{amu}$$

The energy change (ΔE) for the process is

$$\begin{aligned} \Delta E &= (\Delta m)c^2 \\ &= (1.5813 \text{amu})(3.00 \times 10^8 \text{m/s})^2 \\ &= 1.42 \times 10^{17} \frac{\text{amu} \cdot \text{m}^2}{\text{s}^2} \end{aligned}$$

Let's convert to more familiar energy units (J/W atom).

$$\frac{1.42 \times 10^{17} \text{amu} \cdot \text{m}^2}{1 \text{s}^2} \times \frac{1.00 \text{g}}{6.022 \times 10^{23} \text{amu}} \times \frac{1 \text{kg}}{1000 \text{g}} \times \frac{1 \text{J}}{\frac{1 \text{kg} \cdot \text{m}^2}{\text{s}^2}} = 2.36 \times 10^{-10} \text{J}$$

This is the nuclear binding energy. It's the energy required to break up one tungsten-184 nucleus into 74 protons and 110 neutrons.

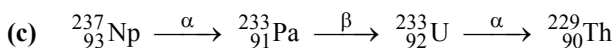
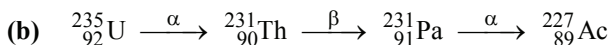
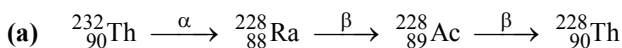
When comparing the stability of any two nuclei we must account for the fact that they have different numbers of nucleons. For this reason, it is more meaningful to use the *nuclear binding energy per nucleon*, defined as

$$\text{nuclear binding energy per nucleon} = \frac{\text{nuclear binding energy}}{\text{number of nucleons}}$$

For the tungsten-184 nucleus,

$$\text{nuclear binding energy per nucleon} = \frac{2.36 \times 10^{-10} \text{ J/W atom}}{184 \text{ nucleons/W atom}} = \mathbf{1.28 \times 10^{-12} \text{ J/nucleon}}$$

23.23 Alpha emission decreases the atomic number by two and the mass number by four. Beta emission increases the atomic number by one and has no effect on the mass number.



23.24 Strategy: According to Equation (13.3) of the text, the number of radioactive nuclei at time zero (N_0) and time t (N_t) is

$$\ln \frac{N_t}{N_0} = -\lambda t$$

and the corresponding half-life of the reaction is given by Equation (13.6) of the text:

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Using the information given in the problem and the first equation above, we can calculate the rate constant, λ . Then, the half-life can be calculated from the rate constant.

Solution: We can use the following equation to calculate the rate constant λ for each point.

$$\ln \frac{N_t}{N_0} = -\lambda t$$

From day 0 to day 1, we have

$$\ln \frac{389}{500} = -\lambda(1 \text{ d})$$

$$\lambda = 0.251 \text{ d}^{-1}$$

Following the same procedure for the other days,

t (d)	mass (g)	λ (d ⁻¹)
0	500	
1	389	0.251
2	303	0.250
3	236	0.250
4	184	0.250
5	143	0.250
6	112	0.249

The average value of λ is **0.250 d⁻¹**.

We use the average value of λ to calculate the half-life.

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{0.250 \text{ d}^{-1}} = \mathbf{2.77 \text{ d}}$$

23.25 The number of atoms decreases by half for each half-life. For ten half-lives we have:

$$(5.00 \times 10^{22} \text{ atoms}) \times \left(\frac{1}{2}\right)^{10} = \mathbf{4.89 \times 10^{19} \text{ atoms}}$$

23.26 Since all radioactive decay processes have first-order rate laws, the decay rate is proportional to the amount of radioisotope at any time. The half-life is given by the following equation:

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} \quad (1)$$

There is also an equation that relates the number of nuclei at time zero (N_0) and time t (N_t).

$$\ln \frac{N_t}{N_0} = -\lambda t$$

We can use this equation to solve for the rate constant, λ . Then, we can substitute λ into Equation (1) to calculate the half-life.

The time interval is:

$$(2:15 \text{ p.m., 12/17/92}) - (1:00 \text{ p.m., 12/3/92}) = 14 \text{ d} + 1 \text{ hr} + 15 \text{ min} = 20,235 \text{ min}$$

$$\ln \left(\frac{2.6 \times 10^4 \text{ dis/min}}{9.8 \times 10^5 \text{ dis/min}} \right) = -\lambda(20,235 \text{ min})$$

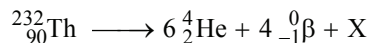
$$\lambda = 1.8 \times 10^{-4} \text{ min}^{-1}$$

Substitute λ into equation (1) to calculate the half-life.

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{1.8 \times 10^{-4} \text{ min}^{-1}} = \mathbf{3.9 \times 10^3 \text{ min or 2.7 d}}$$

23.27 A truly first-order rate law implies that the mechanism is unimolecular; in other words the rate is determined only by the properties of the decaying atom or molecule and does not depend on collisions or interactions with other objects. This is why radioactive dating is reliable.

23.28 The equation for the overall process is:



The final product isotope must be ${}_{82}^{208}\text{Pb}$.

23.29 We start with the integrated first-order rate law, Equation (13.3) of the text:

$$\ln \frac{[\text{A}]_t}{[\text{A}]_0} = -\lambda t$$

We can calculate the rate constant, λ , from the half-life using Equation (13.6) of the text, and then substitute into Equation (13.3) to solve for the time.

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{28.1 \text{ yr}} = 0.0247 \text{ yr}^{-1}$$

Substituting:

$$\ln\left(\frac{0.200}{1.00}\right) = -(0.0247 \text{ yr}^{-1})t$$

$$t = \mathbf{65.2 \text{ yr}}$$

23.30 Let's consider the decay of A first.

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{4.50 \text{ s}} = 0.154 \text{ s}^{-1}$$

Let's convert λ to units of day^{-1} .

$$0.154 \frac{1}{\text{s}} \times \frac{3600 \text{ s}}{1 \text{ h}} \times \frac{24 \text{ h}}{1 \text{ d}} = 1.33 \times 10^4 \text{ d}^{-1}$$

Next, use the first-order rate equation to calculate the amount of A left after 30 days.

$$\ln \frac{[\text{A}]_t}{[\text{A}]_0} = -\lambda t$$

Let x be the amount of A left after 30 days.

$$\ln \frac{x}{100} = -(1.33 \times 10^4 \text{ d}^{-1})(30 \text{ d}) = -3.99 \times 10^5$$

$$\frac{x}{100} = e^{(-3.99 \times 10^5)}$$

$$x \approx 0$$

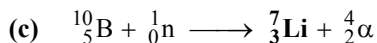
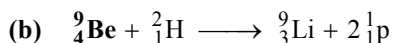
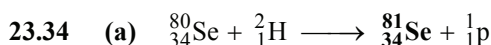
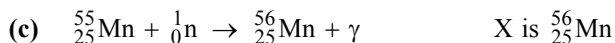
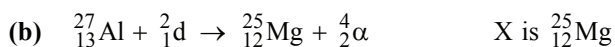
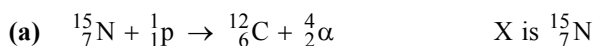
Thus, **no A remains**.

For B: As calculated above, all of A is converted to B in less than 30 days. In fact, essentially all of A is gone in less than 1 day! This means that at the beginning of the 30 day period, there is 1.00 mol of B present. The half life of B is 15 days, so that after two half-lives (30 days), there should be **0.25 mole of B** left.

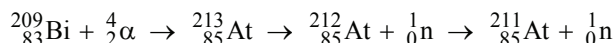
For C: As in the case of A, the half-life of C is also very short. Therefore, at the end of the 30-day period, **no C is left**.

For D: D is not radioactive. 0.75 mol of B reacted in 30 days; therefore, due to a 1:1 mole ratio between B and D, there should be **0.75 mole of D** present after 30 days.

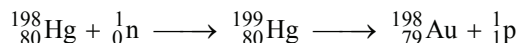
23.33 In the shorthand notation for nuclear reactions, the first symbol inside the parentheses is the "bombarding" particle (reactant) and the second symbol is the "ejected" particle (product).



23.35 All you need is a high-intensity alpha particle emitter. Any heavy element like plutonium or curium will do. Place the bismuth-209 sample next to the alpha emitter and wait. The reaction is:



23.36 Upon bombardment with neutrons, mercury-198 is first converted to mercury-199, which then emits a proton. The reaction is:



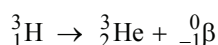
23.47 The easiest experiment would be to add a small amount of aqueous iodide containing some radioactive iodine to a saturated solution of lead(II) iodide. If the equilibrium is dynamic, radioactive iodine will eventually be detected in the solid lead(II) iodide.

Could this technique be used to investigate the forward and reverse rates of this reaction?

23.48 The fact that the radioisotope appears only in the I_2 shows that the IO_3^- is formed only from the IO_4^- . Does this result rule out the possibility that I_2 could be formed from IO_4^- as well? Can you suggest an experiment to answer the question?

- 23.49** On paper, this is a simple experiment. If one were to dope part of a crystal with a radioactive tracer, one could demonstrate diffusion in the solid state by detecting the tracer in a different part of the crystal at a later time. This actually happens with many substances. In fact, in some compounds one type of ion migrates easily while the other remains in fixed position!
- 23.50** Add iron-59 to the person's diet, and allow a few days for the iron-59 isotope to be incorporated into the person's body. Isolate red blood cells from a blood sample and monitor radioactivity from the hemoglobin molecules present in the red blood cells.
- 23.51** The design and operation of a Geiger counter are discussed in Figure 23.18 of the text.
- 23.52** Apparently there is a sort of Pauli exclusion principle for nucleons as well as for electrons. When neutrons pair with neutrons and when protons pair with protons, their spins cancel. Even-even nuclei are the only ones with no net spin.

- 23.53 (a)** The balanced equation is:



- (b)** The number of tritium (T) atoms in 1.00 kg of water is:

$$\begin{aligned} (1.00 \times 10^3 \text{ g H}_2\text{O}) &\times \frac{1 \text{ mol H}_2\text{O}}{18.02 \text{ g H}_2\text{O}} \times \frac{6.022 \times 10^{23} \text{ molecules H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \times \frac{2 \text{ H atoms}}{1 \text{ H}_2\text{O}} \times \frac{1 \text{ T atom}}{1.0 \times 10^{17} \text{ H atoms}} \\ &= 6.68 \times 10^8 \text{ T atoms} \end{aligned}$$

The number of disintegrations per minute will be:

$$\text{rate} = \lambda(\text{number of T atoms}) = \lambda N = \frac{0.693}{t_{1/2}} N$$

$$\text{rate} = \left(\frac{0.693}{12.5 \text{ yr}} \times \frac{1 \text{ yr}}{365 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \right) (6.68 \times 10^8 \text{ T atoms})$$

$$\text{rate} = 70.5 \text{ T atoms/min} = \mathbf{70.5 \text{ disintegrations/min}}$$

- 23.54 (a)** One millicurie represents 3.70×10^7 disintegrations/s. The rate of decay of the isotope is given by the rate law: $\text{rate} = \lambda N$, where N is the number of atoms in the sample. We find the value of λ in units of s^{-1} :

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.20 \times 10^6 \text{ yr}} \times \frac{1 \text{ yr}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 9.99 \times 10^{-15} \text{ s}^{-1}$$

The number of atoms (N) in a 0.500 g sample of neptunium-237 is:

$$0.500 \text{ g} \times \frac{1 \text{ mol}}{237.0 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = 1.27 \times 10^{21} \text{ atoms}$$

$$\begin{aligned} \text{rate of decay} &= \lambda N \\ &= (9.99 \times 10^{-15} \text{ s}^{-1})(1.27 \times 10^{21} \text{ atoms}) = 1.27 \times 10^7 \text{ atoms/s} \end{aligned}$$

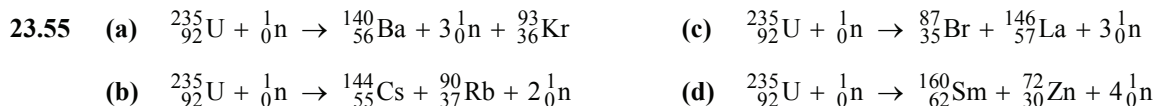
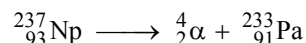
We can also say that:

$$\text{rate of decay} = 1.27 \times 10^7 \text{ disintegrations/s}$$

The activity in millicuries is:

$$(1.27 \times 10^7 \text{ disintegrations/s}) \times \frac{1 \text{ millicurie}}{3.70 \times 10^7 \text{ disintegrations/s}} = \mathbf{0.343 \text{ millicuries}}$$

(b) The decay equation is:



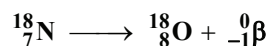
23.56 We use the same procedure as in Problem 23.20.

	Isotope	Atomic Mass (amu)	Nuclear Binding Energy (J/nucleon)
(a)	${}_{10}^{10}\text{B}$	10.0129	1.040×10^{-12}
(b)	${}_{11}^{11}\text{B}$	11.00931	1.111×10^{-12}
(c)	${}_{14}^{14}\text{N}$	14.00307	1.199×10^{-12}
(d)	${}_{26}^{56}\text{Fe}$	55.9349	1.410×10^{-12}

23.57 The balanced nuclear equations are:



23.58 When an isotope is above the belt of stability, the neutron/proton ratio is too high. The only mechanism to correct this situation is beta emission; the process turns a neutron into a proton. Direct neutron emission does not occur.

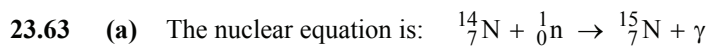
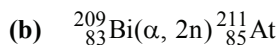
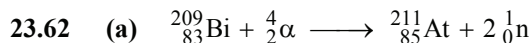


Oxygen-18 is a stable isotope.

23.59 Because both Ca and Sr belong to Group 2A, radioactive strontium that has been ingested into the human body becomes concentrated in bones (replacing Ca) and can damage blood cell production.

23.60 The age of the fossil can be determined by radioactively dating the age of the deposit that contains the fossil.

23.61 Normally the human body concentrates iodine in the thyroid gland. The purpose of the large doses of KI is to displace radioactive iodine from the thyroid and allow its excretion from the body.



(b) X-ray analysis only detects shapes, particularly of metal objects. Bombs can be made in a variety of shapes and sizes and can be constructed of "plastic" explosives. Thermal neutron analysis is much more specific than X-ray analysis. However, articles that are high in nitrogen other than explosives (such as silk, wool, and polyurethane) will give "false positive" test results.

23.64 Because of the relative masses, the force of gravity on the sun is much greater than it is on Earth. Thus the nuclear particles on the sun are already held much closer together than the equivalent nuclear particles on the earth. Less energy (lower temperature) is required on the sun to force fusion collisions between the nuclear particles.

23.65 The neutron-to-proton ratio for tritium equals 2 and is thus outside the belt of stability. In a more elaborate analysis, it can be shown that the decay of tritium to ${}^3\text{He}$ is exothermic; thus, the total energy of the products is less than the reactant.

23.66 Step 1: The half-life of carbon-14 is 5730 years. From the half-life, we can calculate the rate constant, λ .

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

Step 2: The age of the object can now be calculated using the following equation.

$$\ln \frac{N_t}{N_0} = -\lambda t$$

N = the number of radioactive nuclei. In the problem, we are given disintegrations per second per gram. The number of disintegrations is directly proportional to the number of radioactive nuclei. We can write,

$$\ln \frac{\text{decay rate of old sample}}{\text{decay rate of fresh sample}} = -\lambda t$$

$$\ln \frac{0.186 \text{ dps/g C}}{0.260 \text{ dps/g C}} = -(1.21 \times 10^{-4} \text{ yr}^{-1})t$$

$$t = 2.77 \times 10^3 \text{ yr}$$

23.67 $\ln \frac{N_t}{N_0} = -\lambda t$

$$\ln \frac{\text{mass of fresh sample}}{\text{mass of old sample}} = -(1.21 \times 10^{-4} \text{ yr}^{-1})(50000 \text{ yr})$$

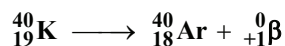
$$\ln \frac{1.0 \text{ g}}{x \text{ g}} = -6.05$$

$$\frac{1.0}{x} = e^{-6.05}$$

$$x = 424$$

$$\text{Percent of C-14 left} = \frac{1.0}{424} \times 100\% = \mathbf{0.24\%}$$

23.68 (a) The balanced equation is:



(b) First, calculate the rate constant λ .

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{1.2 \times 10^9 \text{ yr}} = 5.8 \times 10^{-10} \text{ yr}^{-1}$$

Then, calculate the age of the rock by substituting λ into the following equation. ($N_t = 0.18N_0$)

$$\ln \frac{N_t}{N_0} = -\lambda t$$

$$\ln \frac{0.18}{1.00} = -(5.8 \times 10^{-10} \text{ yr}^{-1})t$$

$$t = \mathbf{3.0 \times 10^9 \text{ yr}}$$

23.69 All isotopes of radium are radioactive; therefore, radium is not naturally occurring and would not be found with barium. However, radium is a decay product of uranium-238, so it is found in uranium ores.

23.70 (a) In the ${}^{90}\text{Sr}$ decay, the mass defect is:

$$\begin{aligned} \Delta m &= (\text{mass } {}^{90}\text{Y} + \text{mass } e^-) - \text{mass } {}^{90}\text{Sr} \\ &= [(89.907152 \text{ amu} + 5.4857 \times 10^{-4} \text{ amu}) - 89.907738 \text{ amu}] = -3.743 \times 10^{-5} \text{ amu} \\ &= (-3.743 \times 10^{-5} \text{ amu}) \times \frac{1 \text{ g}}{6.022 \times 10^{23} \text{ amu}} = -6.216 \times 10^{-29} \text{ g} = -6.216 \times 10^{-32} \text{ kg} \end{aligned}$$

The energy change is given by:

$$\begin{aligned} \Delta E &= (\Delta m)c^2 \\ &= (-6.126 \times 10^{-32} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \\ &= -5.59 \times 10^{-15} \text{ kg m}^2/\text{s}^2 = -5.59 \times 10^{-15} \text{ J} \end{aligned}$$

Similarly, for the ${}^{90}\text{Y}$ decay, we have

$$\begin{aligned} \Delta m &= (\text{mass } {}^{90}\text{Zr} + \text{mass } e^-) - \text{mass } {}^{90}\text{Y} \\ &= [(89.904703 \text{ amu} + 5.4857 \times 10^{-4} \text{ amu}) - 89.907152 \text{ amu}] = -1.900 \times 10^{-3} \text{ amu} \end{aligned}$$

$$= (-1.900 \times 10^{-3} \text{ amu}) \times \frac{1 \text{ g}}{6.022 \times 10^{23} \text{ amu}} = -3.156 \times 10^{-27} \text{ g} = -3.156 \times 10^{-30} \text{ kg}$$

and the energy change is:

$$\Delta E = (-3.156 \times 10^{-30} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = -2.84 \times 10^{-13} \text{ J}$$

The energy released in the above two decays is $5.59 \times 10^{-15} \text{ J}$ and $2.84 \times 10^{-13} \text{ J}$. The total amount of energy released is:

$$(5.59 \times 10^{-15} \text{ J}) + (2.84 \times 10^{-13} \text{ J}) = 2.90 \times 10^{-13} \text{ J}.$$

- (b) This calculation requires that we know the rate constant for the decay. From the half-life, we can calculate λ .

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{28.1 \text{ yr}} = 0.0247 \text{ yr}^{-1}$$

To calculate the number of moles of ^{90}Sr decaying in a year, we apply the following equation:

$$\ln \frac{N_t}{N_0} = -\lambda t$$

$$\ln \frac{x}{1.00} = -(0.0247 \text{ yr}^{-1})(1.00 \text{ yr})$$

where x is the number of moles of ^{90}Sr nuclei left over. Solving, we obtain:

$$x = 0.9756 \text{ mol } ^{90}\text{Sr}$$

Thus the number of moles of nuclei which decay in a year is

$$(1.00 - 0.9756) \text{ mol} = 0.0244 \text{ mol} = \mathbf{0.024 \text{ mol}}$$

This is a reasonable number since it takes 28.1 years for 0.5 mole of ^{90}Sr to decay.

- (c) Since the half-life of ^{90}Y is much shorter than that of ^{90}Sr , we can safely assume that *all* the ^{90}Y formed from ^{90}Sr will be converted to ^{90}Zr . The energy changes calculated in part (a) refer to the decay of individual nuclei. In 0.024 mole, the number of nuclei that have decayed is:

$$0.0244 \text{ mol} \times \frac{6.022 \times 10^{23} \text{ nuclei}}{1 \text{ mol}} = 1.47 \times 10^{22} \text{ nuclei}$$

Realize that there are two decay processes occurring, so we need to add the energy released for each process calculated in part (a). Thus, the heat released from 1 mole of ^{90}Sr waste in a year is given by:

$$\mathbf{\text{heat released}} = (1.47 \times 10^{22} \text{ nuclei}) \times \frac{2.90 \times 10^{-13} \text{ J}}{1 \text{ nucleus}} = \mathbf{4.26 \times 10^9 \text{ J}} = \mathbf{4.26 \times 10^6 \text{ kJ}}$$

This amount is roughly equivalent to the heat generated by burning 50 tons of coal! Although the heat is released slowly during the course of a year, effective ways must be devised to prevent heat damage to the storage containers and subsequent leakage of radioactive material to the surroundings.

23.71 A radioactive isotope with a shorter half-life because more radiation would be emitted over a certain period of time.

23.72 First, let's calculate the number of disintegrations/s to which 7.4 mC corresponds.

$$7.4 \text{ mC} \times \frac{1 \text{ Ci}}{1000 \text{ mC}} \times \frac{3.7 \times 10^{10} \text{ disintegrations/s}}{1 \text{ Ci}} = 2.7 \times 10^8 \text{ disintegrations/s}$$

This is the rate of decay. We can now calculate the number of iodine-131 atoms to which this radioactivity corresponds. First, we calculate the half-life in seconds:

$$t_{\frac{1}{2}} = 8.1 \text{ d} \times \frac{24 \text{ h}}{1 \text{ d}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 7.0 \times 10^5 \text{ s}$$

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} \quad \text{Therefore, } \lambda = \frac{0.693}{7.0 \times 10^5 \text{ s}} = 9.9 \times 10^{-7} \text{ s}^{-1}$$

$$\text{rate} = \lambda N$$

$$2.7 \times 10^8 \text{ disintegrations/s} = (9.9 \times 10^{-7} \text{ s}^{-1})N$$

$$N = 2.7 \times 10^{14} \text{ iodine-131 atoms}$$

23.73 The energy of irradiation is not sufficient to bring about nuclear transmutation.

23.74 One curie represents 3.70×10^{10} disintegrations/s. The rate of decay of the isotope is given by the rate law: $\text{rate} = \lambda N$, where N is the number of atoms in the sample and λ is the first-order rate constant. We find the value of λ in units of s^{-1} :

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{1.6 \times 10^3 \text{ yr}} = 4.3 \times 10^{-4} \text{ yr}^{-1}$$

$$\frac{4.3 \times 10^{-4}}{1 \text{ yr}} \times \frac{1 \text{ yr}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 1.4 \times 10^{-11} \text{ s}^{-1}$$

Now, we can calculate N , the number of Ra atoms in the sample.

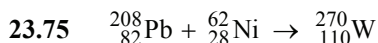
$$\text{rate} = \lambda N$$

$$3.7 \times 10^{10} \text{ disintegrations/s} = (1.4 \times 10^{-11} \text{ s}^{-1})N$$

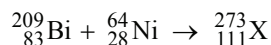
$$N = 2.6 \times 10^{21} \text{ Ra atoms}$$

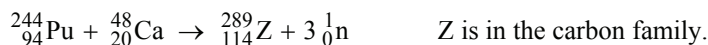
By definition, 1 curie corresponds to exactly 3.7×10^{10} nuclear disintegrations per second which is the decay rate equivalent to that of 1 g of radium. Thus, the mass of 2.6×10^{21} Ra atoms is 1 g.

$$\frac{2.6 \times 10^{21} \text{ Ra atoms}}{1.0 \text{ g Ra}} \times \frac{226.03 \text{ g Ra}}{1 \text{ mol Ra}} = 5.9 \times 10^{23} \text{ atoms/mol} = N_{\text{A}}$$



W and X are transition metals.



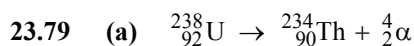


23.76 All except gravitational have a nuclear origin.

23.77 There was radioactive material inside the box.

23.78 U-238, $t_{\frac{1}{2}} = 4.5 \times 10^9$ yr and Th-232, $t_{\frac{1}{2}} = 1.4 \times 10^{10}$ yr.

They are still present because of their long half lives.



$$\Delta m = 234.0436 + 4.0026 - 238.0508 = -0.0046 \text{ amu}$$

$$\Delta E = \Delta mc^2 = (-0.0046 \text{ amu})(3.00 \times 10^8 \text{ m/s})^2 = -4.14 \times 10^{14} \text{ amu}^2/\text{s}^2$$

$$\Delta E = \frac{-4.14 \times 10^{14} \text{ amu} \cdot \text{m}^2}{1 \text{ s}^2} \times \frac{1.00 \text{ kg}}{6.022 \times 10^{26} \text{ amu}} \times \frac{1 \text{ J}}{1 \text{ kg} \cdot \text{m}^2/\text{s}^2} = -6.87 \times 10^{-13} \text{ J}$$

(b) The smaller particle (α) will move away at a greater speed due to its lighter mass.

23.80 $E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} = \frac{(3.00 \times 10^8 \text{ m/s})(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{2.4 \times 10^{-13} \text{ J}} = 8.3 \times 10^{-13} \text{ m} = \mathbf{8.3 \times 10^{-4} \text{ nm}}$$

This wavelength is clearly in the γ -ray region of the electromagnetic spectrum.

23.81 The α particles emitted by ${}^{241}\text{Am}$ ionize the air molecules between the plates. The voltage from the battery makes one plate positive and the other negative, so each plate attracts ions of opposite charge. This creates a current in the circuit attached to the plates. The presence of smoke particles between the plates reduces the current, because the ions that collide with smoke particles (or steam) are usually absorbed (and neutralized) by the particles. This drop in current triggers the alarm.

23.82 Only ${}^3\text{H}$ has a suitable half-life. The other half-lives are either too long or too short to accurately determine the time span of 6 years.

23.83 (a) The nuclear submarine can be submerged for a long period without refueling.

(b) Conventional diesel engines receive an input of oxygen. A nuclear reactor does not.

23.84 Obviously, a small scale chain reaction took place. Copper played the crucial role of reflecting neutrons from the splitting uranium-235 atoms back into the uranium sphere to trigger the chain reaction. Note that a sphere has the most appropriate geometry for such a chain reaction. In fact, during the implosion process prior to an atomic explosion, fragments of uranium-235 are pressed roughly into a sphere for the chain reaction to occur (see Section 23.5 of the text).

- 23.85** From the half-life, we can determine the rate constant, λ . Next, using the first-order integrated rate law, we can calculate the amount of copper remaining. Finally, from the initial amount of Cu and the amount remaining, we can calculate the amount of Zn produced.

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{12.8 \text{ h}} = 0.0541 \text{ h}^{-1}$$

Next, plug the amount of copper, the time, and the rate constant into the first-order integrated rate law, to calculate the amount of copper remaining.

$$\ln \frac{N_t}{N_0} = -\lambda t$$

$$\ln \frac{\text{grams Cu remaining}}{84.0 \text{ g}} = -(0.0541 \text{ h}^{-1})(18.4 \text{ h})$$

$$\frac{\text{grams Cu remaining}}{84.0 \text{ g}} = e^{-(0.0541 \text{ h}^{-1})(18.4 \text{ h})}$$

$$\text{grams Cu remaining} = 31.0 \text{ g}$$

The quantity of Zn produced is:

$$\text{g Zn} = \text{initial g Cu} - \text{g Cu remaining} = 84.0 \text{ g} - 31.0 \text{ g} = \mathbf{53.0 \text{ g Zn}}$$

- 23.86** In this problem, we are asked to calculate the molar mass of a radioactive isotope. Grams of sample are given in the problem, so if we can find moles of sample we can calculate the molar mass. The rate constant can be calculated from the half-life. Then, from the rate of decay and the rate constant, the number of radioactive nuclei can be calculated. The number of radioactive nuclei can be converted to moles.

First, we convert the half-life to units of minutes because the rate is given in dpm (disintegrations per minute). Then, we calculate the rate constant from the half-life.

$$(1.3 \times 10^9 \text{ yr}) \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ h}} = 6.8 \times 10^{14} \text{ min}$$

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{6.8 \times 10^{14} \text{ min}} = 1.0 \times 10^{-15} \text{ min}^{-1}$$

Next, we calculate the number of radioactive nuclei from the rate and the rate constant.

$$\text{rate} = \lambda N$$

$$2.9 \times 10^4 \text{ dpm} = (1.0 \times 10^{-15} \text{ min}^{-1})N$$

$$N = 2.9 \times 10^{19} \text{ nuclei}$$

Convert to moles of nuclei, and then determine the molar mass.

$$(2.9 \times 10^{19} \text{ nuclei}) \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ nuclei}} = 4.8 \times 10^{-5} \text{ mol}$$

$$\text{molar mass} = \frac{\text{g of substance}}{\text{mol of substance}} = \frac{0.0100 \text{ g}}{4.8 \times 10^{-5} \text{ mol}} = 2.1 \times 10^2 \text{ g/mol}$$

23.87 (a) First, we calculate the rate constant using Equation (13.6) of the text.

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{9.50 \text{ min}} = 0.0729 \text{ min}^{-1}$$

Next, we use Equation (13.3) of the text to calculate the number of ^{27}Mg nuclei remaining after 30.0 minutes.

$$\ln \frac{N_t}{N_0} = -\lambda t$$

$$\ln \frac{N_t}{4.20 \times 10^{12}} = -(0.0729 \text{ min}^{-1})(30.0 \text{ min})$$

$$\frac{N_t}{4.20 \times 10^{12}} = e^{-(0.0729 \text{ min}^{-1})(30.0 \text{ min})}$$

$$N_t = 4.71 \times 10^{11} \text{ Mg nuclei remain}$$

(b) The activity (R) is given by

$$R = \frac{\text{number of decays}}{\text{unit time}} = \lambda N$$

We first convert the rate constant to units of s^{-1} .

$$0.0729 \frac{1}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 1.22 \times 10^{-3} \text{ s}^{-1}$$

At $t = 0$,

$$R = (1.22 \times 10^{-3} \text{ s}^{-1})(4.20 \times 10^{12} \text{ nuclei}) = 5.12 \times 10^9 \text{ decays/s}$$

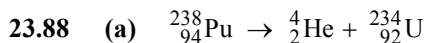
$$(5.12 \times 10^9 \text{ decays/s}) \times \frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ decays/s}} = 0.138 \text{ Ci}$$

At $t = 30.0$ minutes,

$$R = (1.22 \times 10^{-3} \text{ s}^{-1})(4.71 \times 10^{11} \text{ nuclei}) = 5.75 \times 10^8 \text{ decays/s}$$

$$(5.75 \times 10^8 \text{ decays/s}) \times \frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ decays/s}} = 0.0155 \text{ Ci}$$

(c) The probability is just the first-order decay constant, $1.22 \times 10^{-3} \text{ s}^{-1}$. This is valid if the half-life is large compared to one second, which is true in this case.



(b) At $t = 0$, the number of ${}^{238}\text{Pu}$ atoms is

$$(1.0 \times 10^{-3} \text{ g}) \times \frac{1 \text{ mol}}{238 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = 2.53 \times 10^{18} \text{ atoms}$$

The decay rate constant, λ , is

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{86 \text{ yr}} = 0.00806 \frac{1}{\text{yr}} \times \frac{1 \text{ yr}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 2.56 \times 10^{-10} \text{ s}^{-1}$$

$$\text{rate} = \lambda N_0 = (2.56 \times 10^{-10} \text{ s}^{-1})(2.53 \times 10^{18} \text{ atoms}) = 6.48 \times 10^8 \text{ decays/s}$$

$$\text{Power} = (\text{decays/s}) \times (\text{energy/decay})$$

$$\text{Power} = (6.48 \times 10^8 \text{ decays/s})(9.0 \times 10^{-13} \text{ J/decay}) = 5.8 \times 10^{-4} \text{ J/s} = \mathbf{5.8 \times 10^{-4} \text{ W} = 0.58 \text{ mW}}$$

At $t = 10 \text{ yr}$,

$$\text{Power} = (0.58 \text{ mW})(0.92) = \mathbf{0.53 \text{ mW}}$$

23.89 (a) The volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Volume is proportional to the number of nucleons. Therefore,

$$V \propto A \text{ (mass number)}$$

$$r^3 \propto A$$

$$r = r_0 A^{1/3}, \text{ where } r_0 \text{ is a proportionality constant.}$$

(b) We can calculate the volume of the ${}^{238}\text{U}$ nucleus by substituting the equation derived in part (a) into the equation for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3 A$$

$$V = \frac{4}{3}\pi(1.2 \times 10^{-15} \text{ m})^3(238) = \mathbf{1.7 \times 10^{-42} \text{ m}^3}$$

23.90 $1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s}$

Let R_0 be the activity of the injected $20.0 \text{ mCi } {}^{99m}\text{Tc}$.

$$R_0 = (20.0 \times 10^{-3} \text{ Ci}) \times \frac{3.70 \times 10^{10} \text{ decays/s}}{1 \text{ Ci}} = 7.4 \times 10^8 \text{ decays/s}$$

$R_0 = \lambda N_0$, where N_0 = number of ${}^{99m}\text{Tc}$ nuclei present.

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{6.0 \text{ h}} = 0.1155 \frac{1}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 3.208 \times 10^{-5} \text{ s}^{-1}$$

$$N_0 = \frac{R_0}{\lambda} = \frac{7.4 \times 10^8 \text{ decays/s}}{3.208 \times 10^{-5} / \text{s}} = 2.307 \times 10^{13} \text{ decays} = 2.307 \times 10^{13} \text{ nuclei}$$

Each of the nuclei emits a photon of energy $2.29 \times 10^{-14} \text{ J}$. The total energy absorbed by the patient is

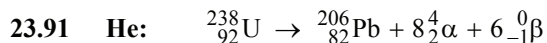
$$E = \frac{2}{3} (2.307 \times 10^{13} \text{ nuclei}) \times \left(\frac{2.29 \times 10^{-14} \text{ J}}{1 \text{ nuclei}} \right) = \mathbf{0.352 \text{ J}}$$

The rad is:

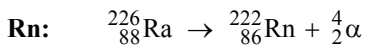
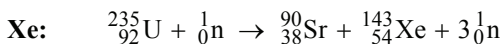
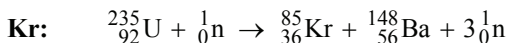
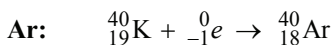
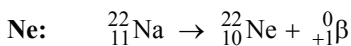
$$\frac{0.352 \text{ J}/10^{-2} \text{ J}}{70} = \mathbf{0.503 \text{ rad}}$$

Given that RBE = 0.98, the rem is:

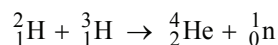
$$(0.503)(0.98) = \mathbf{0.49 \text{ rem}}$$



The reaction represents the overall process for the decay of U-238. See Table 23.3 of the text. α particles produced are eventually converted to helium atoms.



23.92 The ignition of a fission bomb requires an ample supply of neutrons. In addition to the normal neutron source placed in the bomb, the high temperature attained during the chain reaction causes a small scale nuclear fusion between deuterium and tritium.



The additional neutrons produced will enhance the efficiency of the chain reaction and result in a more powerful bomb.